3 Yr. Degree/4 Yr. Honours 1st Semester Examination, 2023 (CCFUP)

Subject : Mathematics

Course : MATH1011 (MAJOR)

(Calculus, Geometry & Vector Calculus)

Time: 3 Hours **Full Marks: 60**

The figures in the margin indicate full marks.

Candidates are required to give their answers in their own words as far as practicable.

Notation and symbols have their usual meaning.

1. Answer *any ten* of the following questions: 2×10=20

(a) Determine the asymptotes of $x^3 + y^3 = 3ax^2$.

(b) Evaluate $\lim_{x\to 0} (\cos x)^{\cot^2 x}$.

(c) If $\lim_{x\to 0} \frac{\sin 2x + a \sin x}{x^3}$ be finite, find the value of 'a' and the value of limit.

(d) If $y^{\frac{1}{m}} + y^{-\frac{1}{m}} = 2x$, prove that $(x^2 - 1)y_2 + xy_1 - m^2 y = 0$.

(e) Evaluate $\int_0^{\frac{\pi}{2}} \sin^7 x \, dx$.

(f) If $I_n = \int_0^{\frac{\pi}{4}} \tan^n x \, dx$, then show that $I_n + I_{n-2} = \frac{1}{n-1}$.

(g) Find the parametric equation of the astroid $x^{\frac{2}{3}} + y^{\frac{2}{3}} = a^{\frac{2}{3}}$.

(h) Find the equation of the tangent plane to the sphere $x^2 + y^2 + z^2 = 5$ at (2, 0, 1).

(i) Find the equation of the cone whose vertex is the origin and the base is the circle $x = a, y^2 + z^2 = b^2$.

(j) Show that the semilatus rectum of a conic is the harmonic mean between the segments of a focal chord.

(k) Find the nature of the conicoid $3x^2 - 2y^2 - 12x - 12y - 6z = 0$.

(l) If $\vec{e}_1$ and $\vec{e}_2$ are unit vectors and θ be the angle between them, then show that $|\vec{e}_1 - \vec{e}_2| = 2 \sin\frac{\theta}{2}$.

(m) Show that $[\vec{\alpha} + \vec{\beta}, \vec{\beta} + \vec{\gamma}, \vec{\gamma} + \vec{\alpha}] = 2\,[\vec{\alpha}, \vec{\beta}, \vec{\gamma}]$, where $\vec{\alpha}, \vec{\beta}, \vec{\gamma}$ are any three vectors.

(n) Give an example to show that a continuous vector valued function may not be differentiable.

(o) If $\vec{r} = a\cos t\,\hat{i} + a \sin t\,\hat{j} + at \tan\alpha\,\hat{k}$, then find $\left[\frac{d\vec{r}}{dt}, \frac{d^2\vec{r}}{dt^2}, \frac{d^3\vec{r}}{dt^3}\right]$.

2. Answer *any four* of the following questions: 5×4=20

(a) Reduce the equation $4x^2 + 4xy + y^2 - 4x - 2y + \lambda = 0$, to the canonical form and determine the nature of the conic for different values of λ.

(b) State and prove Leibnitz's theorem for nth derivative of the product of two functions of the same single variable.

(c) If $x^{\frac{2}{3}} + y^{\frac{2}{3}} = c^{\frac{2}{3}}$ is the envelope of $\frac{x}{a} + \frac{y}{b} = 1$ where a, b are variable parameters and c is a constant, then prove that $a^2 + b^2 = c^2$.

(d) Determine the entire surface area formed by the revolution of the upper half of the cardioid $r = a(1 + \cos\theta)$ about the initial line.

(e) Find the equation and the type of the conic which passes through (–2, 0), touches the y-axis at the origin and has its centre at (1, 1).

(f) Show that the vector function $\vec{f}(t)$ has constant direction if and only if $\vec{f} \times \frac{d\vec{f}}{dt} = \vec{0}$. 2½×2

3. Answer *any two* of the following questions: 10×2=20

(a) (i) Find the all asymptotes of the curve $x^3 + 2x^2y - xy^2 - 2y^3 + xy - y^2 - 1 = 0$.

(ii) Show that the condition that the straight line $\frac{l}{r} = a\cos\theta + b\sin\theta$ may touch the circle $r = 2k\cos\theta$ is $b^2k^2 + 2ak = 1$. 5+5

(b) (i) If $I_{m,n} = \int \sin^m x \cos^n x \, dx$, where $m, n \in N - \{1\}$, then show that $I_{m,n} = \frac{\sin^{m+1} x \cos^{n-1} x}{m+n} + \frac{n-1}{m+n} I_{m,n-2}$.

(ii) Find the length of the arc of the parabola $y^2 = 16x$ measured from the vertex to an extremity of the latus rectum. 5+5

(c) (i) Reduce $6y^2 - 18yz - 6zx + 2xy - 9x + 5y - 5z + 2 = 0$ in canonical form and state the nature of the surface represented by it.

(ii) Show that the condition that the plane $ax + by + cz = 0$ may cut the cone $yz + zx + xy = 0$ in perpendicular lines is $\frac{1}{a} + \frac{1}{b} + \frac{1}{c} = 0$. 5+5

(d) (i) Show that the vectors $\vec{a} \times (\vec{b} \times \vec{c}), \vec{b} \times (\vec{c} \times \vec{a}), \vec{c} \times (\vec{a} \times \vec{b})$ are coplanar.

(ii) Prove that curl grad $f = 0$ for scalar point function f.

(iii) Show that div curl $\vec{f} = 0$ for vector point function $\vec{f}$. 4+3+3

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3 Yr. Degree/4 Yr. Honours 1st Semester Examination, 2023 (CCFUP)

Subject : Mathematics

Course : MATH1021 (MINOR)

(Calculus, Geometry & Vector Calculus)

Time: 3 Hours **Full Marks: 60**

The figures in the margin indicate full marks.

Candidates are required to give their answers in their own words as far as practicable.

Notation and symbols have their usual meaning.

1. Answer *any ten* of the following questions: 2×10=20

(a) Find the surface area of a sphere generated by revolution of a circle $x^2+y^2 = a^2$ about x axis.

(b) Evaluate $\lim_{x\to 0} \frac{x\cos x - \log(1+x)}{x^2}$.

(c) Find the asymptotes parallel to axes of the curve $(x^2 + y^2)x - ay^2 = 0$.

(d) Find the envelope of $y^2 = m^2(x - m)$, m being the parameter.

(e) Obtain the reduction formula for $\int x^n e^{ax}\, dx$, n being a positive integer.

(f) Evaluate $\int_0^1 x^4 \sqrt{1 - x^2}\, dx$.

(g) Evaluate $\int_0^{\frac{\pi}{2}} \cos^5 x\, dx$.

(h) Find the points on $\frac{8}{r} = 3 - \sqrt{2}\cos\theta$ whose radius vector is 4.

(i) Find the nature of the conic $\frac{5}{r} = 2 - 2\cos\theta$ and find its semilatus rectum.

(j) Determine the centre of the conic $3x^2 + 4y^2 - 12x + 8y + 4 = 0$.

(k) Find the centre and the radius of the sphere $3x^2 + 3y^2 + 3z^2 + 2x - 4y - 2z - 1 = 0$.

(l) Show that $[\vec{i} - \vec{j}, \vec{j} - \vec{k}, \vec{k} - \vec{i}] = 0$.

(m) If $\vec{r} = \sin t\, \vec{i} + \cos t\, \vec{j} + 2\vec{k}$, then show that $\left|\frac{d^2\vec{r}}{dt^2}\right| = 1$.

(n) Examine whether the vectors $7\vec{i} - 9\vec{j} + 11\vec{k}, 3\vec{i} + \vec{j} - 5\vec{k}, 5\vec{i} - 21\vec{j} + 37\vec{k}$ are coplanar.

(o) Show that $\nabla^2\left(\frac{1}{r}\right) = 0$, where $r = \sqrt{x^2 + y^2 + z^2}$.

2. Answer *any four* of the following questions: 5×4=20

(a) If $y = e^{m\sin^{-1}x}$, establish $(1 - x^2)y_{n+2} - (2n + 1)x\, y_{n+1} - (n^2 + m^2)y_n = 0$, symbols used are usual meaning.

(b) Determine $\lim_{x\to 0}\left(\frac{1}{x^2}-\frac{1}{\sin^2 x}\right)$.

(c) Show that the arc of the upper half of the cardioid $r = a\,(1-\cos\theta)$ is bisected at $\theta = \frac{2\pi}{3}$. Show also that the perimeter of the curve is $8a$. 4+1

(d) Let PSP' be a focal chord of a conic $\frac{l}{r} = 1 - e\cos\theta$. Prove that the angle between the tangents at P and P' is $\tan^{-1}\frac{2e\sin\alpha}{1-e^2}$, where α is the angle between the chord and the major axis.

(e) Find the equation of the cylinder, whose generators are parallel to $\frac{x}{-1}=\frac{y}{2}=\frac{z}{3}$ and whose guiding curve is $x^2+y^2=9, z=1$.

(f) Find div $\vec{F}$ and curl $\vec{F}$ where $\vec{F} = \text{grad}\,(x^3+y^3+z^3-3xyz)$.

3. Answer *any two* of the following questions: 10×2=20

(a) (i) Find the asymptotes of $y^3-6xy^2+11x^2y-6x^3+x+y=0$.

(ii) Find the values of a and b in order that $\lim_{x\to 0}\frac{x(1+a\cos x)-b\sin x}{x^3}$ may equal to 1. 6+4

(b) (i) If $I_n = \int \tan^n x\,dx$, where $n \in \mathbb{N}-\{1\}$, then show that $I_n = \frac{\tan^{n-1}x}{n-1} - I_{n-2}$. Hence find $\int \tan^4 x\,dx$.

(ii) Find the length of one arc of the cycloid $x = a\,(\theta-\sin\theta), y = a\,(1-\cos\theta)$. (4+2)+4

(c) (i) Reduce the equation $3x^2+2xy+3y^2-16x+20=0$ in canonical form and then determine its nature.

(ii) Find polar equation of the tangent to the conic $\frac{2}{r} = 1-\cos\theta$ at $\theta = \frac{1}{2}\pi$. 6+4

(d) (i) If $\vec{r} = 3t\,\vec{i}+3t^2\vec{j}+2t^3\vec{k}$, then find the value of $\left[\frac{d\vec{r}}{dt}, \frac{d^2\vec{r}}{dt^2}, \frac{d^3\vec{r}}{dt^3}\right]$.

(ii) If $\vec{f}$ be a continuously differentiable vector valued function, then show that div curl $\vec{f} = 0$.

(iii) If $\varphi(x,y,z) = x^2yz+4xz^2$, then find grad φ at $(1,-2,-1)$.

(iv) If $\vec{r} = \sin t\,\vec{i}-\cos t\,\vec{j}-\vec{k}$ and $\vec{s} = \cos t\,\vec{i}+\sin t\,\vec{j}+t\,\vec{k}$, find $\frac{d}{dt}(\vec{r}\times\vec{s})$. 3+3+2+2

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3 Yr. Degree/4 Yr. Honours 1st Semester Examination, 2023 (CCFUP)

Subject : Mathematics

Course : MATH1051 (SEC)

(Graph Theory)

Time: 2 Hours **Full Marks: 40**

The figures in the margin indicate full marks.

Candidates are required to give their answers in their own words as far as practicable.

Notation and symbols have their usual meaning.

1. Answer *any five* questions: 2×5=10

(a) How many vertices are there in a 4-regular graph with 10 edges?

(b) If G is a graph of order p and size q and has t vertices of degree m and all other vertices of degree n, then show that $(m-n)t + pn = 2q$.

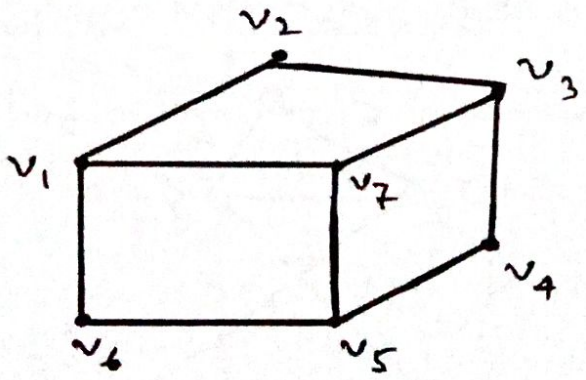
(c) Let G be a connected graph with 8 vertices and 7 edges. Does G contain a vertex of degree 1? Justify your answer.

(d) Show that for any complete graph k_p with p vertices and any vertex v of k_p, $k_p - v = k_{p-1}$ holds.

(e) Let G be a simple connected planar graph with 13 vertices and 19 edges. Then find the number of faces in the planar embedding of the graph.

(f) Define an Eulerian graph and give necessary condition for a graph to be Eulerian.

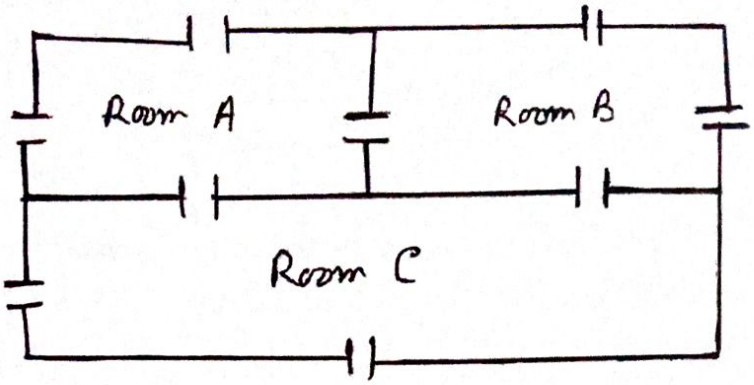
(g) Examine whether the following simple graph is bipartite or not. Give reason.



(h) Does there exist a tree of order 15 such that the sum of the degrees of the vertices is 30? Justify.

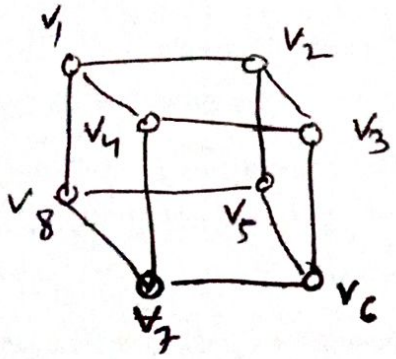
2. Answer *any two* questions: 5×2=10

(a) Consider the floor plan of the three-room flat with doors as shown below:

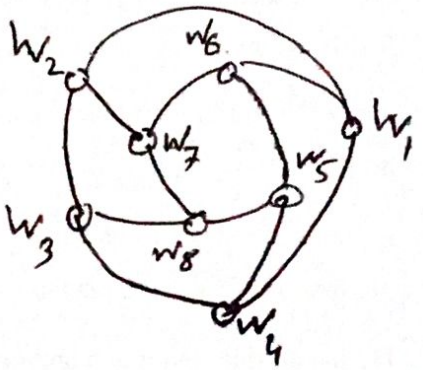


Each room is connected by doors with every other room and outside as shown in the plan. Is it possible to start in a room or outside and take a walk that goes through each door exactly once? If there is such a walk, find it. Can you return to the starting point through this walk? Justify. 4+1

(b) When are two graphs said to be isomorphic? Show that the following two graphs G_1 and G_2 are isomorphic:

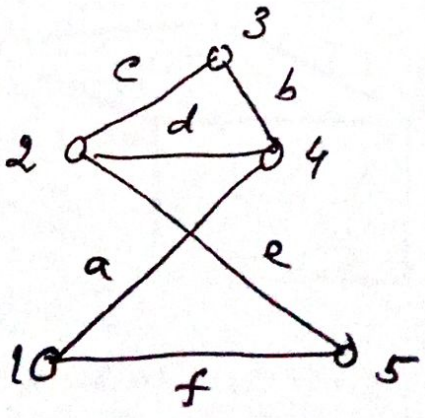


G_1



G_2 1+4

(c) (i) Represent the following graph with an incidence matrix:

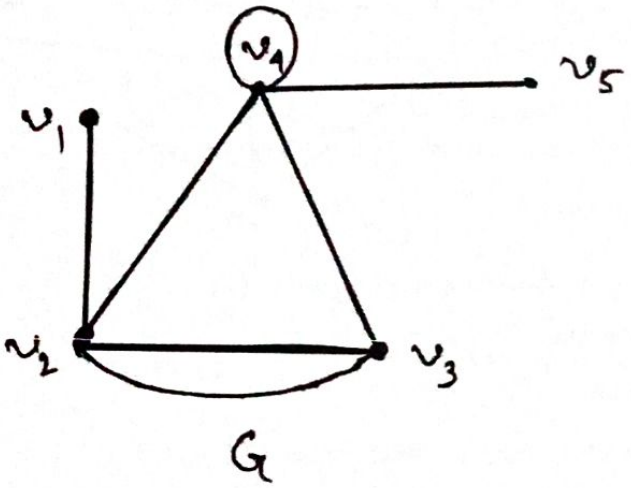


(ii) Draw the digraph G corresponding to the adjacency matrix:

$$A(G) = \begin{pmatrix} 0 & 0 & 1 & 1 \\ 0 & 0 & 1 & 0 \\ 1 & 1 & 0 & 1 \\ 1 & 1 & 1 & 0 \end{pmatrix}$$

3+2

(d) The pictorial representation of a graph G is given below:



(i) Find the degree sequence of the graph G.

(ii) Find the pendant vertices in G, if exists.

(iii) Is the graph G regular? 2+2+1

3. Answer *any two* questions: 10×2=20

(a) (i) Prove that a graph with n vertices is a tree if and only if it is acyclic and contains exactly $(n-1)$ edges.

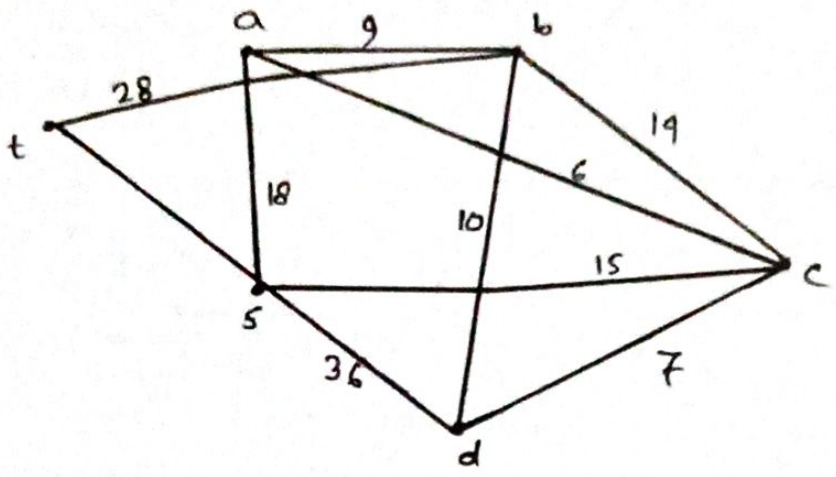
(ii) State and prove Euler's theorem on planar graphs. 4+6

(b) (i) Find an optional Hamiltonian cycle in a directed network whose weight matrix A is given below:

$$A = \begin{pmatrix} - & 17 & 10 & 15 & 17 \\ 18 & - & 6 & 10 & 20 \\ 12 & 5 & - & 14 & 19 \\ 12 & 11 & 15 & - & 7 \\ 16 & 31 & 18 & 6 & - \end{pmatrix}$$

(ii) Prove that any self-complementary graph has $4n$ or $4n + 1$ vertices. 6+4

(c) (i) Applying Dijkstra's algorithm, find a shortest path between the vertices s and t in the following weighted graph G.



G

(ii) Find the smallest positive integer n such that the complete graph K_n has at least 400 edges.

(iii) Decide whether the degree sequence {7, 6, 4, 3, 3, 3, 2, 1} is graphical or not. 5+3+2

(d) (i) In a group of nine persons, is it possible for each person to shake hands with exactly five other persons? Justify.

(ii) Draw a graph which is Eulerian but not Hamiltonian. Justify your answer.

(iii) Discuss the concept of graph colouring and its applications. 3+3+4